### Efficient TCTL Model Checking Algorithm for Timed Actos

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# Analysis Support

- Floating Time Transition System for Schedulability and Deadlock-Freedom analysis
- Transforming to Erlang to simulate
- Event-Based Property Analysis using Simulation Engine of Timed Rebeca
- TCTL verification of Timed Rebeca models



#### Difficulties of TCTL Model Checking

- There is no efficient algorithm
- Only a subset of TCTL can be check efficiently
- Does having discrete time make it easier?
  - Duration transition graph (DTG) is a good alternative

#### Semantics Of Timed Rebeca in Timed Transition System

- The tuple  $TTS = (S, s_0, Act, \rightarrow, AP, L)$  is the timed transition system of a Timed Rebeca model where:
  - S is a set of states  $\left(\prod_{i \in I} (V_{s,i}, B_{s,i}, pc_{s,i}, res_{s,i}), now_s\right)$
  - Act is a set of actions  $\bigcup_{i \in I} ((I \times i \times \mathcal{M}_i) \times \mathbb{N} \times \mathbb{N}) \cup \{\tau\} \cup \mathbb{N}$
  - Three types of transitions
    - Taking an event
    - Internal transition
    - progress of time

#### Simple Timed Rebeca Model

```
reactiveclass RC1 (3) {
knownrebecs {
 RC2 r2;
RC1() {
 self.m1();
msgsrv m1() {
 delay(2);
 r2.m2();
 delay(2);
 r2.m3();
 self.m1() after (10);
```

reactiveclass RC2 (4) { knownrebecs { RC1 r1; RC2() { } msgsrv m2() { } msgsrv m3() { } main { RC1 r1(r2):(); RC2 r2(r1):();

#### Simple Timed Rebeca Model



	_	queue	$[(r1 \rightarrow r1. m1(), 0, \infty)]$				
	2	рс	-				
		queue	-				
	r2	рс	-				
$(r1 \rightarrow r1 m1()) 0 q$							
	(/1 //1.m1(),0,						
	S1						
		queue -					
	7	nc	m1·2				
		queue	-				
	2	nc	-				
		pc					
time = 2 $time = time + 2$							
	*						
			52				
	_	queue	-				
	12	рс	<i>m</i> 1:2				
		queue	-				
	2	рс	-				
			$\tau(r1)$				
			+				
	<b>S</b> 2						
		queue	-				
	1	nc	m1:4				
		pc .	$[(r1 \rightarrow r2 m2() 0 m)]$				
	5	queue	$[(1 \rightarrow 12.m2(), 0, \infty)]$				
	-	pc	-				
			$(r1 \rightarrow r2. m2(), 0, \infty)$				
			+				
			S4				
		queue					
	1	nc	m1·4				
		queue					
	2	queue	-				
	_	DC	-				
			time - time 1.2				
time = 4			time = time + 2				
time = 4			time = time + 2				
time = 4			time = time + 2				
time = 4	-	queue	time = time + 2 \$5				
time = 4	11	queue	time = time + 2 \$5 - m1: 4				
time = 4	r1	queue pc queue	time = time + 2 \$5 - m1: 4 -				
time = 4	r2 r1	queue pc queue pc	time = time + 2 55 - m1: 4 -				
time = 4	r2 r1	queue pc queue pc	time = time + 2 55 - m1: 4 - -				
time = 4	r2 r1	queue pc queue pc	time = time + 2 55 - m1:4 - - τ(r1)				
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time = 4	r1 r2 r1	queue pc queue pc queue	time = time + 2 55 - m1: 4 - - $\tau(r1)$ 56 -				
time = 4	r1 r2 r1	queue pc queue pc queue pc queue	time = time + 2 55 - m1: 4 - - τ(r1) 56 - [(r1 → r2, m3(), 0, ∞)]				
time = 4	r2 r1 r2 r1	queue pc queue pc queue pc queue pc queue pc	time = time + 2 time = time + 2 rime = time + 2				
time = 4	r2 r1 r2 r1	queue pc queue pc queue pc queue pc	time = time + 2 55 m1:4 $\tau(r1)$ 56 $[(r1 \rightarrow r2.m3(), 0, \infty)]$				
time = 4	r2 r1 r2 r1	queue pc queue pc queue pc queue pc	time = time + 2 55 - m1:4 - - $\tau(r1)$ 56 - $[(r1 \rightarrow r2.m3(), 0, \infty)]$ -				
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time = 4	r2 r1 r2 r1	queue pc queue pc queue pc queue pc queue	$time = time + 2$ $55$ - m1:4 $\tau(r1)$ $56$ - [(r1 → r2.m3(),0,∞)] - (r1 → r2.m3(),0,∞) S7 -				
time = 4	r1 r2 r1 r2 r1	queue pc queue pc queue pc queue pc queue pc	$time = time + 2$ $55$ - $m1:4$ $\tau(r1)$ $56$ [(r1 $\rightarrow$ r2.m3(),0, $\infty$ )] - (r1 $\rightarrow$ r2.m3(),0, $\infty$				
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time = 4	12 11 12 11	queue pc queue pc queue pc queue pc queue pc queue pc	$time = time + 2$ $55$ - $m1: 4$ $\tau(r1)$ $56$ - $[(r1 \rightarrow r2. m3(), 0, \infty)]$ - $(r1 \rightarrow r2. m3(), 0, \infty]$				
time = 4 time = 14	r2 r1 r2 r1 r2 r1	queue pc queue pc queue pc queue pc queue pc queue pc	$time = time + 2$ $s5$ - $m1: 4$ $\tau(r1)$ $s6$ - $[(r1 \rightarrow r2. m3(), 0, \infty)]$ - $(r1 \rightarrow r2. m3(), 0, \infty]$				
time = 4 time = 14	12 11 12 11 12 11 12 11	queue pc queue pc queue pc queue pc queue pc queue pc	time = time + 2				
time = 4 time = 14	1 r2 r1 r2 r1 r2 r1 r2 r1	queue pc queue pc queue pc queue pc queue pc queue pc	$time = time + 2$ $time = time + 2$ $rime = time + 2$ $r(r1)$ $r(r1)$ $r(r1)$ $r(r1 \rightarrow r2.m3(),0,\infty)$				
time = 4	r1 r2 r1 r2 r1 r2 r1 r2 r1	queue pc queue pc queue pc queue pc queue pc queue pc	$time = time + 2$ $time = time + 2$ $rime = time + 2$ $r(r1)$ $r(r1)$ $r(r1)$ $r(r1 \rightarrow r2.m3(),0,\infty)$				
time = 4 time = 14	1         12         11         12         11         12         11	queue           pc           queue           pc	$time = time + 2$ $time = time + 2$ $rime = time + 2$ $r(r1)$ $r(r1)$ $r(r1)$ $r(r1 \rightarrow r2.m3(),0,\infty)$				
time = 4	r2 r1 r2 r1 r2 r1 r2 r1 r2 r1	queue           pc           queue           pc	$time = time + 2$ $time = time + 2$ $rime = time + 2$ $r(r1)$ $r(r1)$ $r(r1)$ $r(r1 \rightarrow r2.m3(),0,\infty)$				



















### Duration Transition Graph



 $S_1$ 

# TCTL Model Checking for DTG

- There is a polynomial time model checking algorithm for TCTL<> properties but model checking of TCTL= is NP-Complete
- Combination of CTL model checking and shortest/ longest path search
  - Try to find a set of states which satisfy a given TCTL formula without any timed constraint
  - Check timed constraints

# How the Algorithm Works for $\exists \Phi_1 \mathbf{U}^{\leq \mathbf{c}} \Phi_2$

- Assume  $DTG_{\mathcal{M}}^{sub} = (S', s'_0, Act, \rightarrow', AP', L')$  as a reduced version of  $DTG_{\mathcal{M}}$  which satisfies  $\exists \Phi_1 U \Phi_2$
- Any state s in  $DTG^{sub}_{\mathcal{M}}$  satisfies the time version of the formula if and only if there is path from s to state s' such that the length of path is less than the time constraint.
- This can be checked using O(n<sup>2</sup>) algorithm

#### TCTL<sub><></sub> Model Checking of Timed Rebeca Models

- TTS of a Timed Rebeca model is a DTG
  - All the transitions are assumed to be internal transitions (no action label) with zero time duration
  - Progress-of-Time transitions are assumed as transitions with tight duration
- Model checking Timed Rebeca models against TCTL<sub><></sub> properties is possible in O(n<sup>2</sup>)

#### A New Reduction Technique

- There are some transient states in the state space (Residual time is zero in transient states)
- Transient behavior is not interested in some systems
  - It is possible to eliminate transient states from the state space
  - The overhead of reduction must be smaller than the gain of model checking on smaller state space

#### Example of How Reduction Technique Works



#### Cost Free Reduction Technique!

- Timed Rebeca models must be checked to be Zeno free
- It can be checked by using DFS search to detect cycles without progress-of-time states
  - Timed model is discrete in Timed Rebeca

```
Algorithm 1: ZenoFree(s) analyzes the model for Zeno-
 freedom.
   Input: State s of a timed transition system T
   Output: The part of T reachable from s is Zeno-free or
             not
1 visited \leftarrow \emptyset
2 forall the state s' ∈ Successors(s) do
       if s' \notin visited then
 3
           visited \leftarrow visited \cup \{s'\}
 4
           recStack(s') \leftarrow true
5
           if ZenoFree(s') = false then
6
               return false
7
           recStack(s') \leftarrow false
8
       else
9
           if recStack(s') = true \land now(s') = now(s) then
10
                return false
11
12 return true
```

#### Cost Free Reduction Technique!

- Reduction technique requires BFS traversal to figure out the set of next level progress-of-time states of each state
- Combination of BFS and DFS is required
  - Explore the states among two consequent progress-of-time states by bounded-DFS
  - Then go to the next level

**Algorithm 2:** *BoundedZenoCheck(s)* makes sure that there is no cycle among reachable states from *s* to *npts(s)*. Also sets the nearest progress-of-time states of all the reachable states from *s* to *npts(s)*.

Input: State s of a timed transition system Output: The bounded reachable part of the transition system is Zeno-free or not 1 visited  $\leftarrow 0$ 2 forall the state s' ∈ Successors(s) do if  $s' \notin visited$  then 3 visited  $\leftarrow$  visited  $\cup \{s'\}$ 4 if s' is progress-of-time then 5 //DFS has reached one of its boundaries  $npts(s) \leftarrow npts(s) \cup \{s'\}$ else 7 if now(s) = now(s') then 8  $recStack(s') \leftarrow true$ 9  $childsNPTS \leftarrow BoundedZenoCheck(s')$ 10 $recStack(s') \leftarrow false$ if  $childsNPTS = \emptyset$  then 11 return Ø 12 else 13  $npts(s) \leftarrow npts(s) \cup childsNPTS$ 14 else 15 //Back-edge is detected 16  $npts(s) \leftarrow npts(s) \cup npts(s')$ else 17 if recStack(s') = true then  $\mathbf{18}$ //There is cycle which shows Zeno behavior 19 return Ø 20 return npts(s)

**Algorithm 3:**  $FTS(TTS_M)$  creates the corresponding FTS of a given TTS or returns  $\emptyset$  in the case of Zeno behavior in the model.

Input: Timed transition system  $TTS_{\mathcal{M}} = (S, s_0, Act, \rightarrow, AP, L)$ Output: Folded timed transition system of M  $1 S' \leftarrow \{s_0\}$ 2 Act'  $\leftarrow \emptyset$  $3 \hookrightarrow \leftarrow \emptyset$  $A AP' \leftarrow AP$ 5  $L' \leftarrow L$ 6 openBorderStates ←  $\{s_0\}$ 7 nextLevelStates  $\leftarrow \emptyset$ 8 repeat while openBorderStates  $\neq \emptyset$  do 9 remove s from openBorderStates 10 $NPTS \leftarrow BoundedZenoCheck(s)$ 11 if  $NPTS = \emptyset$  then 12 return Ø 13 else 14  $nextLevelStates \leftarrow nextLevelStates \cup NPTS$ 15  $S' \leftarrow S' \cup NPTS$ 16 foreach  $s' \in NPTS$  do 17  $\hookrightarrow \leftarrow \hookrightarrow \cup \{(s, act', s')\}$ 18  $Act' \leftarrow Act' \cup \{act'\}$ 19  $openBorderStates \leftarrow nextLevelStates$ 20  $nextLevelStates \leftarrow \emptyset$ 21 **22 until** openBorderStates  $\neq \emptyset$ 23 return  $(S', s_0, Act', \hookrightarrow, AP, L)$ 15

#### More Than Smaller State Space

- Using reduction technique, we are able to efficiently check the model for TCTL<sub>=</sub> properties
- There is a pseudo-polynomial algorithm for finding exact path among two nodes of the graph
  - Preprocessing a graph to uniforms the weights of edges by an O(W<sup>2</sup> n<sup>3</sup>) algorithm
  - Looking for exact path by an O(|k| min{|k|,w} n<sup>2</sup>) algorithm

#### Polynomial Algorithm for Model Checking of TCTL<sub>=</sub> Properties

- All the weights in TTS are positive integers, so there is no need for an  $O(W^2 n^3)$  relaxation algorithm
- In model checking problem, "k" related to the maximum bound number which is used in TCTL<sub>=</sub> formula
  - In many cases "k" can be assumed as an small constant
  - The complexity of finding exact path is reduced to  $O(|k|^2 n^2) = O(n^2)$

# Experimental Results

- Four different models are used
- 90% reduction in the state space size in some cases

Problem	Size	State Space Size	Reduced State	Percentage of
			Space Size	Reduction
	2 customers	77	10	87%
	3 customers	360	39	89%
	4 customers	1825	184	90%
<b>Ticket Service</b>	5 customers	10708	1045	90%
	6 customers	73461	6996	90%
	7 customers	581962	54019	91%
WSAN	-	1920	818	57%
Yarn	-	533	172	68%
8x8 NoC	-	74192	6068	92%

## Conclusion

- We can model check Timed Rebeca models against TCTL<> properties in polynomial time.
- A combination of checking for Zeno freedom and reduction technique is proposed
  - Reducing the state space size without overhead
- We propose an approach which works for model checking of wide range of TCTL<sub>=</sub> formulas in polynomial time

Thank you