Informed search: *best-first search*

informed: use a heuristic estimate of the distance from a node to a goal given by predicate eval/2

```
search_best([Goal|RestAgenda],Goal):-
   goal(Goal).
search_best([CurrentNode|RestAgenda],Goal):-
   children(CurrentNode,Children),
   add_best(Children,RestAgenda,NewAgenda),
   search_best(NewAgenda,Goal).

add_best([],Agenda,Agenda).
add_best([Node|Nodes],Agenda,NewAgenda):-
  insert(Node,Agenda,TmpAgenda),
  add_best(Nodes,TmpAgenda,NewAgenda).
```

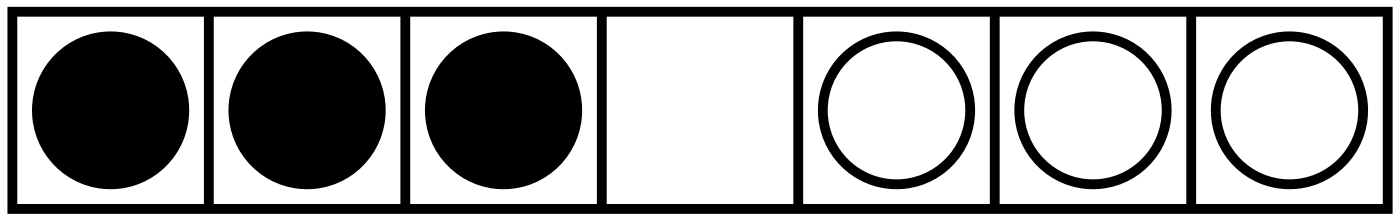
best-first: children of node are added according to heuristic (lowest value first)

Agenda is sorted

add_best(A,B,C): C contains the elements of A and B (B and C sorted according to eval/2)

```
insert(Node,Agenda,NewAgenda):-
   eval(Node,Value),
   insert(Value,Node,Agenda,NewAgenda).
insert(Value,Node, [], [Node]).
insert(Value,Node, [FirstNode|RestOfAgenda], [Node,FirstNode|RestOfAgenda]):-
   eval(FirstNode, FirstNodeValue),
   Value < FirstNodeValue.
insert(Value,Node, [FirstNode|RestOfAgenda], [FirstNode|NewRestOfAgenda]):-
   eval(FirstNode,FirstNodeValue),
   Value >= FirstNodeValue,
   insert(Value,Node,RestOfAgenda,NewRestOfAgenda).
```

Informed search:
best-first search on a puzzle

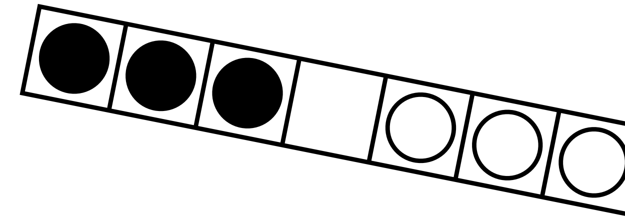


A tile may be moved to the empty spot if there are at most 2 tiles between it and the empty spot.

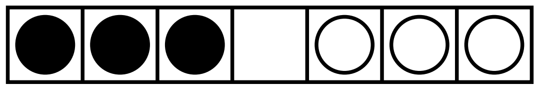
Find a series of moves that bring all the black tiles to the right of all the white tiles.

Cost of a move: 1 if no tiles were in between, otherwise amount of tiles jumped over.

Informed search:
best-first search on a puzzle - encoding



Board:



```
[b,b,b,e,w,w,w]
```

```
get_tile(Position,N,Tile) :-
  get_tile(Position,1,N,Tile).

get_tile([Tile|Tiles],N,N,Tile).
get_tile([Tile|Tiles],N0,N,FoundTile) :-
  N1 is N0+1,
  get_tile(Tiles, N1, N, FoundTile).
```

```
replace([Tile|Tiles],1,ReplacementTile, [ReplacementTile|Tiles]).
replace([Tile|Tiles],N,ReplacementTile, [Tile|RestOfTiles]):-
  N>1,
  N1 is N-1,
  replace(Tiles,N1,ReplacementTile,RestOfTiles).
```

Moves:

```
start_move(move(noparent, [b,b,b,e,w,w,w],0))
```

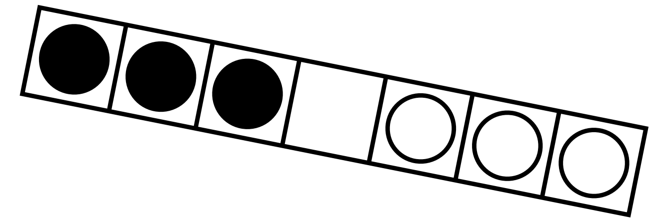
from — to — cost

Agenda items:

```
move_value(Move, Value)
```

heuristic evaluation of position reached by Move

Informed search:
best-first search on a puzzle - algorithm



```
tiles(ListOfPositions, TotalCost):-
  start_move(StartMove),
  eval(StartMove, Value),
  tiles([move_value(StartMove, Value)], FinalMove, [], VisitedMoves),
  order_moves(FinalMove, VisitedMoves, [], ListOfPositions,0, TotalCost).
```

acc for VisitedMoves

best-first search accumulating path

print path backwards from final move to start move

acc for ListOfPositions

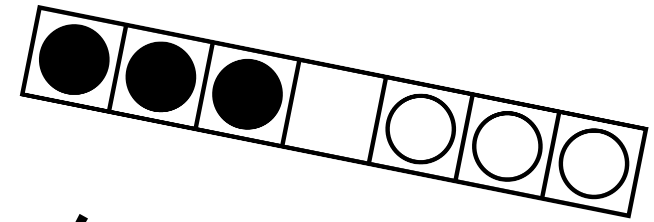
acc for TotalCost

tiles(Agenda, LastMove, V0, V): goal can be reached from a move in Agenda where LastMove is the last move leading to the goal, and V is V0 + the set of moves tried.

```
tiles([move_value(LastMove,Value)|RestAgenda],LastMove,VisitedMoves,VisitedMoves):-
  goal(LastMove).
tiles([move_value(Move,Value)|RestAgenda],Goal,VisitedMoves, FinalVisitedMoves):-
  show_move(Move,Value),
  setof0(move_value(NextMove,NextValue),
         (next_move(Move,NextMove),eval(NextMove,NextValue)),
         Children),
  merge(Children,RestAgenda,NewAgenda),
  tiles(NewAgenda,Goal, [Move|VisitedMoves],FinalVisitedMoves).
```

finds sorted list of children with their evaluation

Informed search:
best-first search on a puzzle - encoding'



```
next_move(move(Position,LastPosition,LastCost),
          move(LastPosition,NewPosition,Cost)) :-
  get_tile(LastPosition, Ne, e),
  get_tile(LastPosition, Nbw, BW),
  not(BW=e),
  Diff is abs(Ne-Nbw),
  Diff<4,
  replace(LastPosition,Ne,BW,IntermediatePosition),
  replace(IntermediatePosition,Nbw,e,NewPosition),
  (Diff=1 -> Cost=1
   ; otherwise -> Cost is Diff-1
  ).
```

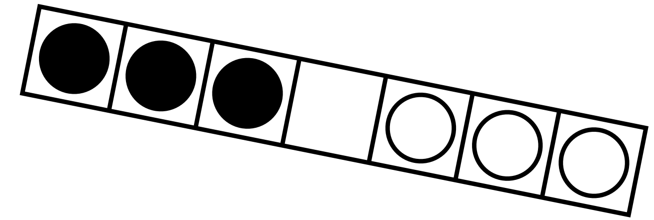
NewPosition is reached in one move from LastPosition with cost Cost

```
goal(Move):-
  eval(Move,0).

eval(move(OldPosition,Position,C),Value):-
  bLeftOfw(Position,Value).

bLeftOfw(Pos,Val):-
  findall((Nb,Nw),
          (get_tile(Pos,Nb,b),get_tile(Pos,Nw,w), Nb<Nw),L),
  length(L,Val).
```

sum of the number of black tiles to the left of each white tile

Informed search:

best-first search on a puzzle - auxiliaries



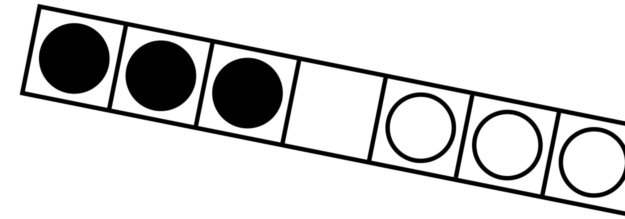
order_moves(FinalMove, VisitedMoves,Positions,FinalPositions,TotalCost,FinalTotalCost):

FinalPositions = Positions + connecting sequence of target positions from VisitedMoves ending in FinalMove’s target position.

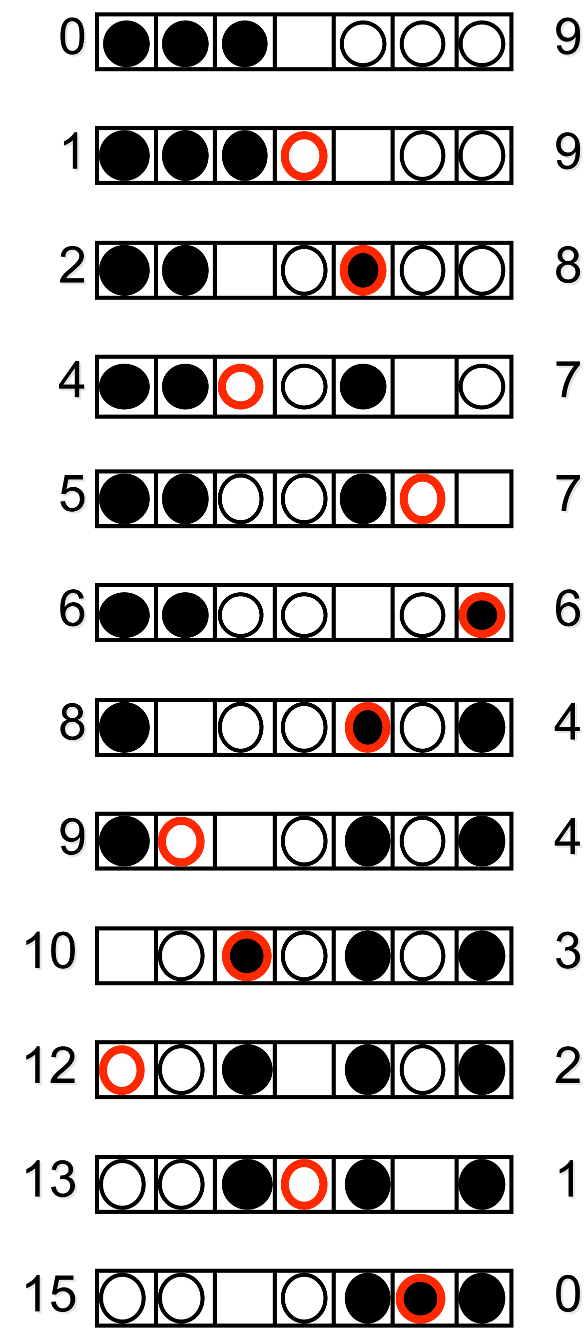
FinalTotalCost = TotalCost + total cost of moves added to Positions to obtain FinalPositions.

```
order_moves(move(noparent,StartPosition,0),
            VisitedMoves,Positions,
            [StartPositionPositions],TotalCost,TotalCost).

order_moves(move(FromPosition,ToPosition,Cost),
            VisitedMoves,Positions,
            FinalPositions,TotalCost,FinalTotalCost):-
  element(PreviousMove, VisitedMoves),
  PreviousMove = move(PreviousPosition, FromPosition,CostOfPreviousMove),
  NewTotalCost is TotalCost + Cost,
  order_moves(PreviousMove,VisitedMoves,
              [ToPosition|Positions],FinalPositions,NewTotalCost,FinalTotalCost).
```

Informed search:
best-first search on a puzzle - example run



```
?- tiles(M,C).
[b,b,b,e,w,w,w]-9
[b,b,b,w,e,w,w]-9
[b,b,e,w,b,w,w]-8
[b,b,w,w,b,e,w]-7
[b,b,w,w,b,w,e]-7
[b,b,w,w,e,w,b]-6
[b,e,w,w,b,w,b]-4
[b,w,e,w,b,w,b]-4
[e,w,b,w,b,w,b]-3
[w,w,b,e,b,w,b]-2
[w,w,b,w,b,e,b]-1

M =[[b,b,b,e,w,w,w], [b,b,b,w,e,w,w],
    [b,b,e,w,b,w,w], [b,b,w,w,b,e,w],
    [b,b,w,w,b,w,e], [b,b,w,w,e,w,b],
    [b,e,w,w,b,w,b], [b,w,e,w,b,w,b],
    [e,w,b,w,b,w,b], [w,w,b,e,b,w,b],
    [w,w,b,w,b,e,b], [w,w,e,w,b,b,b]]

C = 15
```



Informed search:
optimal best search

Best-first search is not complete by itself:

a heuristic might consistently assign lower values to the nodes on an infinite path

An A algorithm is a complete best-first search algorithm that aims at minimizing the total cost along a path from start to goal.

$$f(n) = g(n) + h(n)$$

actual cost so far:
adds breadth-first flavor

estimate on further cost to reach goal:
if optimistic (underestimating the cost), an optimal path will always be found. Such an algorithm is called A*.

h(n)=0 :
degenerates to breadth-first